1888. *Proposed by Alex Aguado, Duke University, Durham, NC.*

Let $A \subseteq X$ be a subset of a topological space, and let $N(A)$ denote the number of sets obtained from A by alternately taking closures and complements (in any order). It is well known that $N(A)$ is at most 14. However, for exactly which $r \leq 14$ is it possible to find A and X such that $N(A) = r$?

Solution by Mark Bowron, Laughlin, NV.
The possible values of r are $\{1, 2, 4, 6, 8, 10, 12, 14\}$. The following sets A_r of real numbers satisfy $N(A_r) = r$ for $r \in \{2, 4, 6, 8, 10, 12, 14\}$: $A_2 = \varnothing$, $A_4 = (-\infty, 1)$, $A_6 = A_4 \cup (1, 2)$, $A_8 = A_6 \cup \{2\}$, $A_{10} = A_8 \cup \{3\}$, $A_{12} = A_{10} \cup [Q \cap (2, 3)]$, $A_{14} = A_{12} \cup \{4\}$. Since no subset of a nonempty topological space can equal its own complement, then $N(A)$ can be odd only if $A = X = \varnothing$. In this case, $N(A) = 1$.

Editor's Note. The fact that $N(A)$ is at most 14 is known as Kuratowski's Closure-Complement Problem.

Also solved by George Apostolopoulos (Greece), Bruce S. Burdick, Robert Calcaterra, Jaime Gutierrez, Victor Pambuccian, and the proposer.