1898. *Proposed by Mark Bowron, Laughlin, NV.*

A subset E of a topological space X is called a *Kuratowski* 14-*set* if 14 distinct sets can be obtained by repeatedly applying closure and complement to E in some order. It is known that Kuratowski 14-sets E with $|E| = 3$ exist. Do any exist with $|E| < 3$?

Solution by Bruce S. Burdick, Roger Williams University, Bristol, RI.
The answer is no. Suppose we had such an E. Using c for closure and i for interior, the sets E, cE, icE, $cicE$, iE, ciE, and $iciE$ must all be distinct. (The other seven sets are the complements of these. Note that $iE = X \setminus c(X \setminus E)$.)

The set iE must not be E and it must not be empty, since that would imply that $ciE = iE$. So that rules out $|E| < 2$. Assume then that $E = \{x, y\}$ and $iE = \{x\}$. Note that if $icE \subseteq ciE$, then it would follow that $icE = iciE$. So there is some $z \in icE$ with $z \notin ciE = c\{x\}$. But $z \in cE$, so it must be that $z \in c\{y\}$. Therefore, $y \in icE$. Since x is isolated, we have $E \subseteq icE$, hence $cE \subseteq cicE$. Therefore, $cE = cicE$, a contradiction.

Also solved by Alex Aguado, George Apostolopoulos (Greece), Jeffrey Boerner, Robert Calcaterra, José H. Nieto (Venezuela), and the proposer. There was one incomplete submission.